

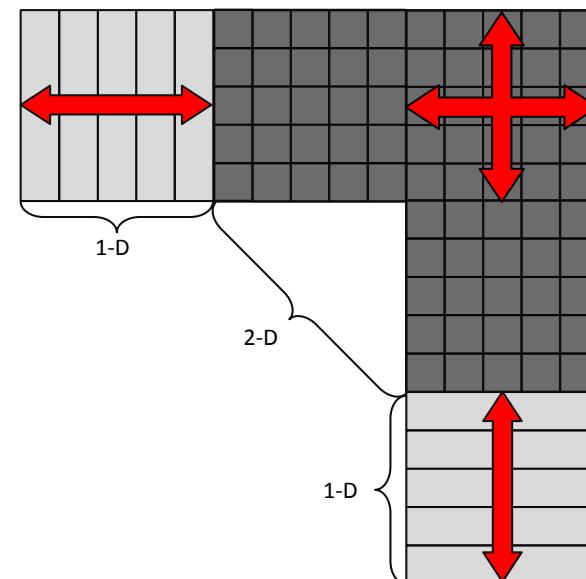


Transient simulation of a pneumatic sharp edged L-shaped pipe

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Motivation

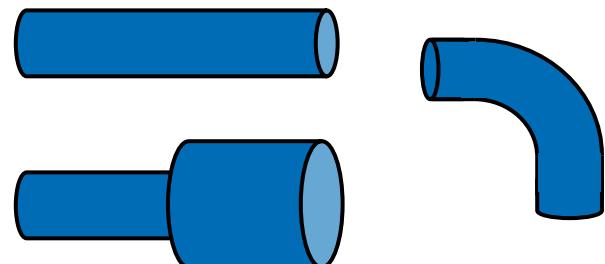
System Prediction

- Increased dynamics in pneumatic systems
- Predictive Maintenance
- System prediction
- Time efficient system simulation



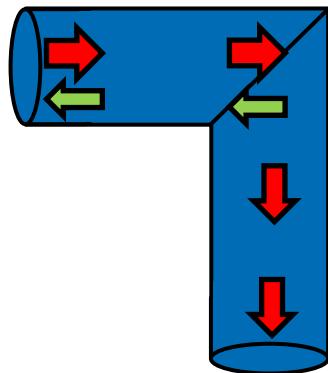
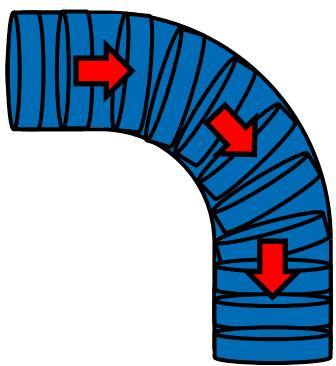
Transient System Simulation

- Component library
- Basic pneumatic components
- One dimensional Riemann Solvers
- Limitation even for simple geometry



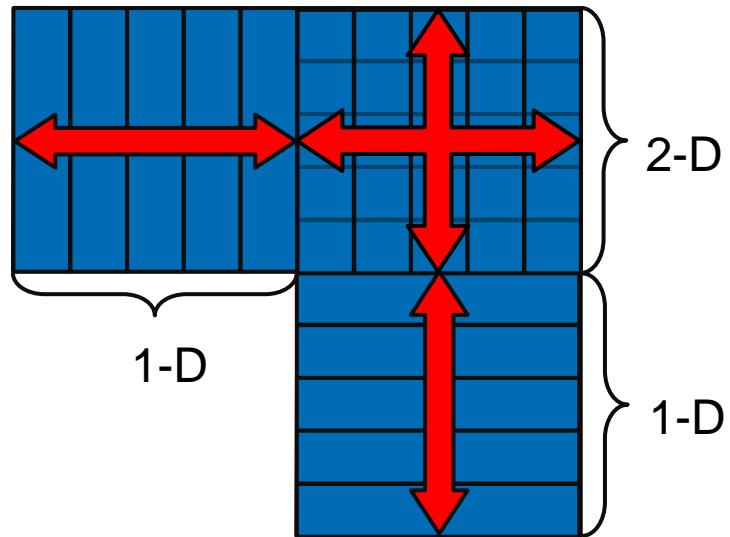
Sharp Edged Elbow

Smooth & Sharp Elbow



- Smooth elbow without reflection
- Sharp elbow with partial reflection
- Reflection in one dimensional solver not possible yet

1-D & 2-D Resolution



- Two dimensional solver needed
- Coupling between 1-D and 2-D required

1 Finite Volume Fundamentals and Approach

2 Analytical and Simulative Validation

3 Conclusion and Outlook

Fundamentals of Finite Volume Method

Fundamental Equations

Navier-Stokes-Equations



Euler-Equations

Mass
Momentum
Energy

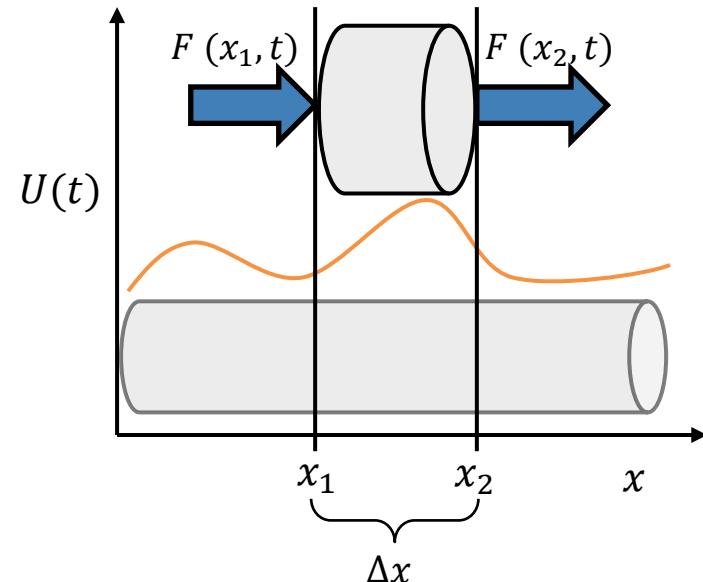
$$\begin{pmatrix} \rho \\ \rho v \\ E \end{pmatrix}_t + \begin{pmatrix} \rho v \\ \rho v^2 + p \\ v(E + p) \end{pmatrix}_x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial U}{\partial t} = - \frac{\partial F}{\partial x}$$

Variable U

Flux $F(U)$

Integration in space



Integration within an arbitrary volume Δx

$$\int_{x_1}^{x_2} \frac{\partial U(x, t)}{\partial t} dx = -F(x_2, t) + F(x_1, t)$$

Fundamentals of Finite Volume Method

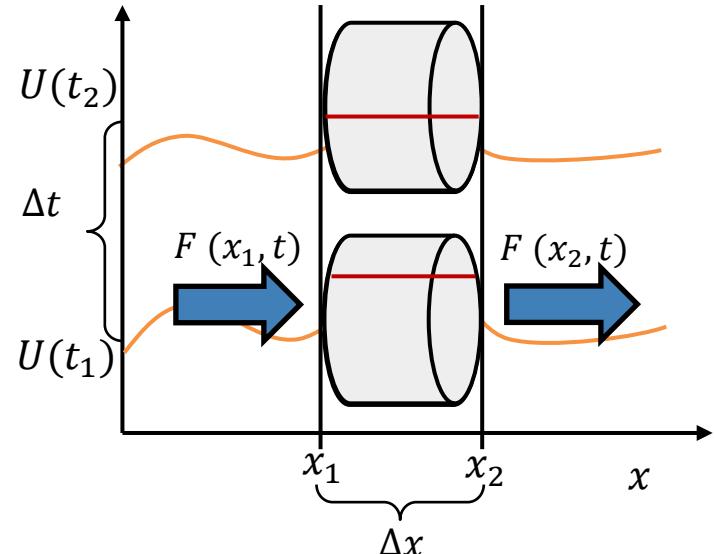
Integration in time

$$\underbrace{\int_{t_1}^{t_2} \int_{x_1}^{x_2} \frac{\partial U(x, t)}{\partial t} dx dt}_{\textcircled{1}} = - \underbrace{\int_{t_1}^{t_2} (F(x_2, t) - F(x_1, t)) dt}_{\textcircled{2}}$$

$$\textcircled{1} \quad \int_{x_1}^{x_2} U(x, t_2) dx - \int_{x_1}^{x_2} U(x, t_1) dx \quad | \div \Delta x$$

Fluxes are integrated in time numerically

$$\textcircled{2} \quad \Delta t (F(x_2, t) - F(x_1, t)) \quad | \div \Delta x$$

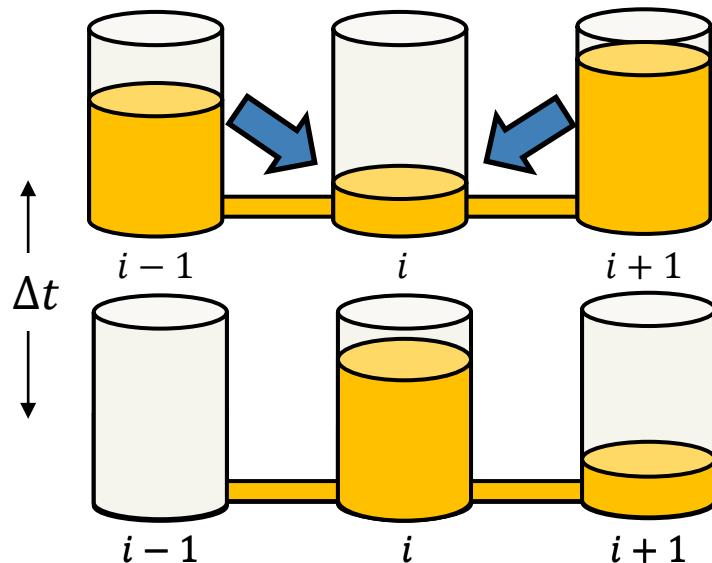


$$\left. \begin{array}{l} \textcircled{1} \quad \tilde{U}(x_{\overline{2,1}}, t_2) - \tilde{U}(x_{\overline{2,1}}, t_1) \\ \textcircled{2} \quad (F(x_2, t) - F(x_1, t)) \frac{\Delta t}{\Delta x} \end{array} \right\} \text{Numerical error due to } \Delta t \text{ possible!}$$

$$\tilde{U}(x_{\overline{2,1}}, t_2) = \tilde{U}(x_{\overline{2,1}}, t_1) - \frac{\Delta t}{\Delta x} (F(x_2, t) - F(x_1, t))$$

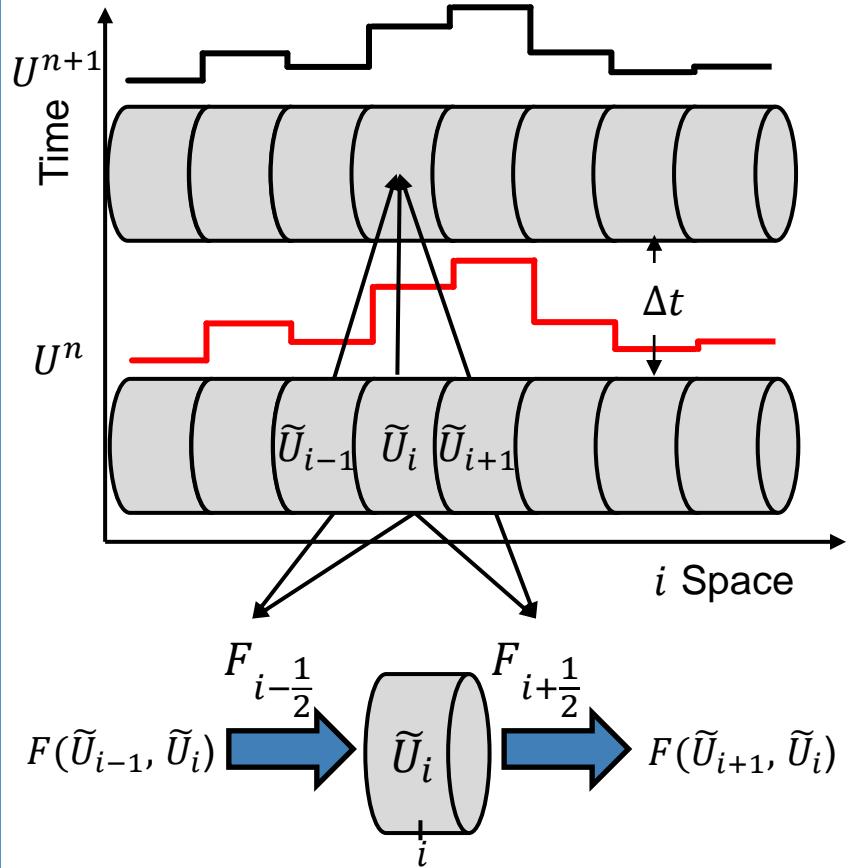
CFL-Criteria

$$\tilde{U}_i^{n+1} = \tilde{U}_i^n - \frac{\Delta t}{\Delta x} (F_{i+\frac{1}{2}} - F_{i-\frac{1}{2}})$$



waiter's time $\Delta t < \frac{\Delta x}{c}$ $\begin{matrix} \text{glas volume} \\ \text{drinking speed} \end{matrix}$

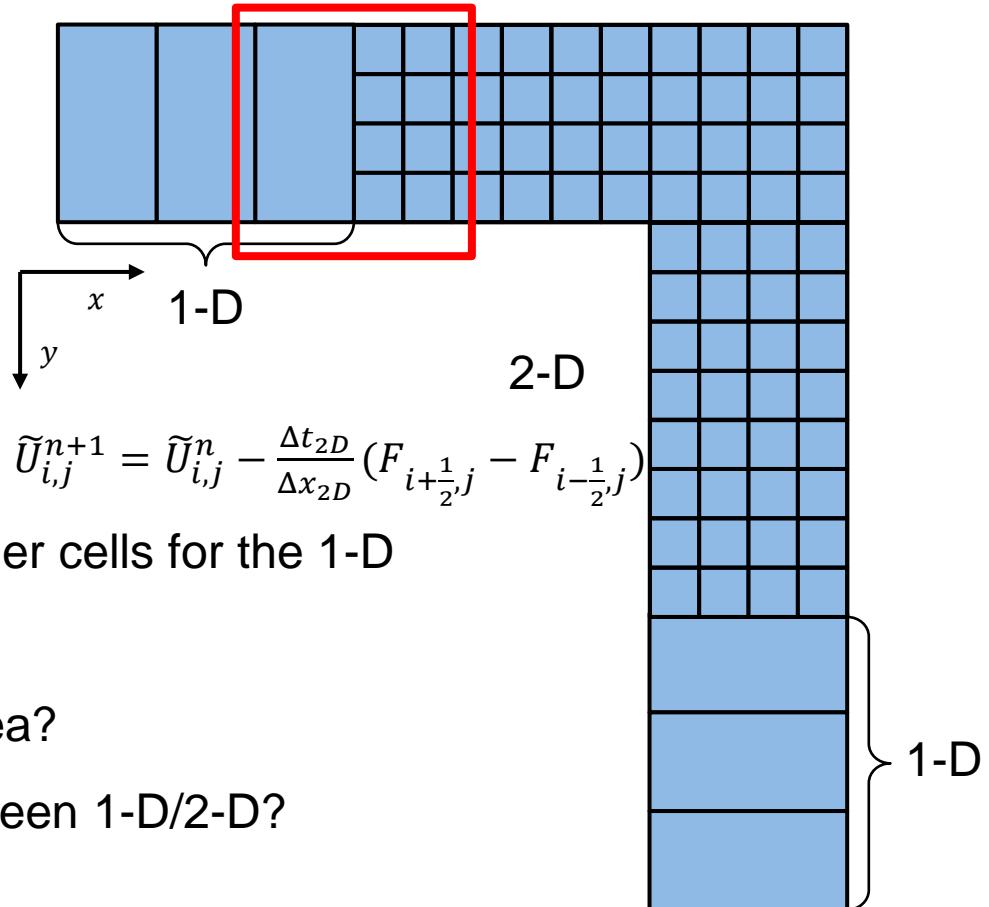
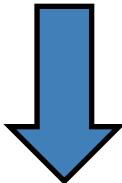
Flux Definition



Two Dimensional Elbow

Coupling 2-D with 1-D elements

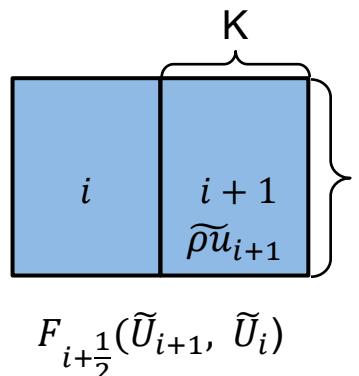
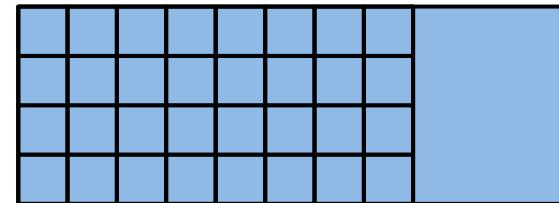
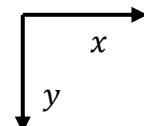
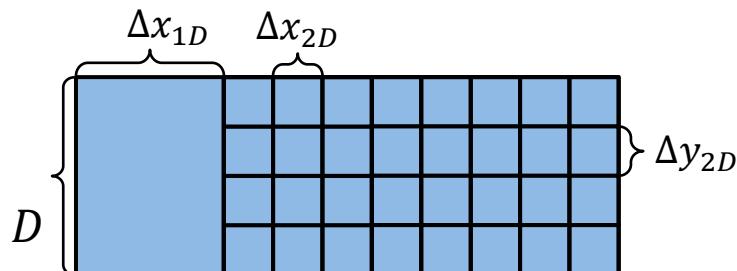
Coupling for given Δt in 1-D elements



1. How to average the 2-D border cells for the 1-D elements?
2. How to discretise the 2-D area?
3. How to build the Fluxes between 1-D/2-D?

Two Dimensional Elbow

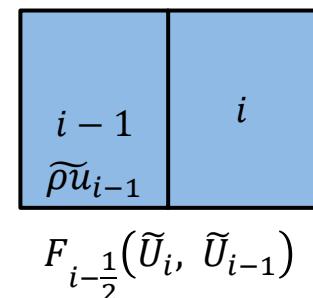
Idea: Conservative Averaging



1: Δx_{2D} fits exactly K times into Δx_{1D}

2: Δy_{2D} fits exactly L times in D

$$\tilde{\rho} \tilde{u} \Delta x D = \Delta x_{2D} \Delta y_{2D} \left(\sum_{i=1}^{i=K} \sum_{j=1}^{j=L} (\rho u_{i,j}) + \left| \sum_{i=1}^{i=K} \sum_{j=1}^{j=L} (\rho v_{i,j}) \right| \right)$$



$$F_{i-\frac{1}{2}}(\tilde{U}_i, \tilde{U}_{i-1})$$

Averaging and mesh generation

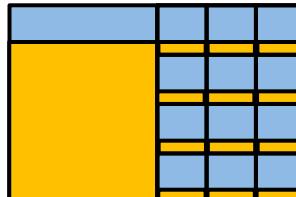
Two Dimensional Elbow

Problem 2-D Solver's border

Smaller volume and greater c

$$\Delta t_{2D} < \frac{\Delta x_{1D}}{c_{2D} K} \quad \rightarrow \quad \Delta t_{2D} = \frac{1}{2} \frac{\Delta t_{1D}}{K}$$

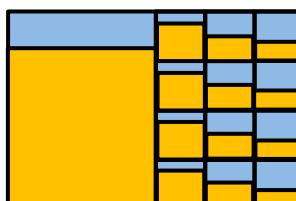
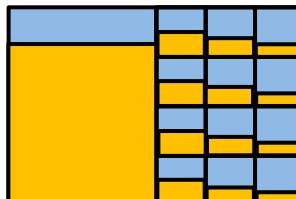
t_1



$$t_1 + \frac{1}{2} \Delta t_{1D}$$



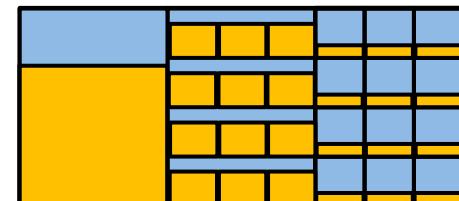
$$t_1 + \Delta t_{1D}$$



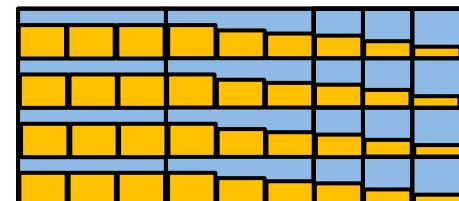
Idea: Buffer Cell+Dynamic Mesh

Additional buffer cell

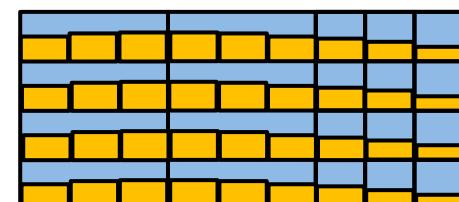
t_1



$$t_1 + \frac{1}{2} \Delta t_{1D}$$



$$t_1 + \Delta t_{1D}$$



1 Finite Volume Fundamentals and Approach

2 Analytical and Simulative Validation

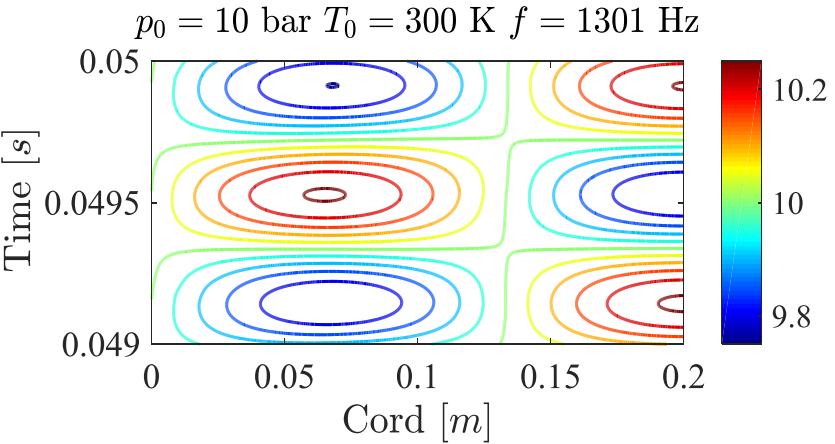
3 Conclusion and Outlook

Acoustic Theory Validation

1D Pipe

Closed end standing wave

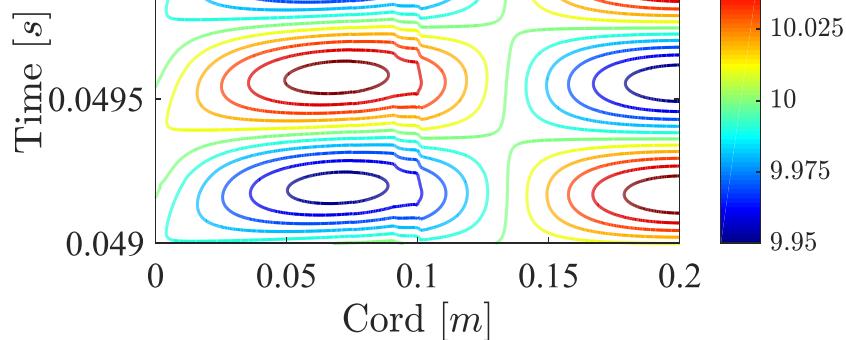
$$f = \frac{3a}{4L}$$



2D Elbow

Closed end standing wave

$$p_0 = 10 \text{ bar } T_0 = 300 \text{ K } f = 1301 \text{ Hz}$$

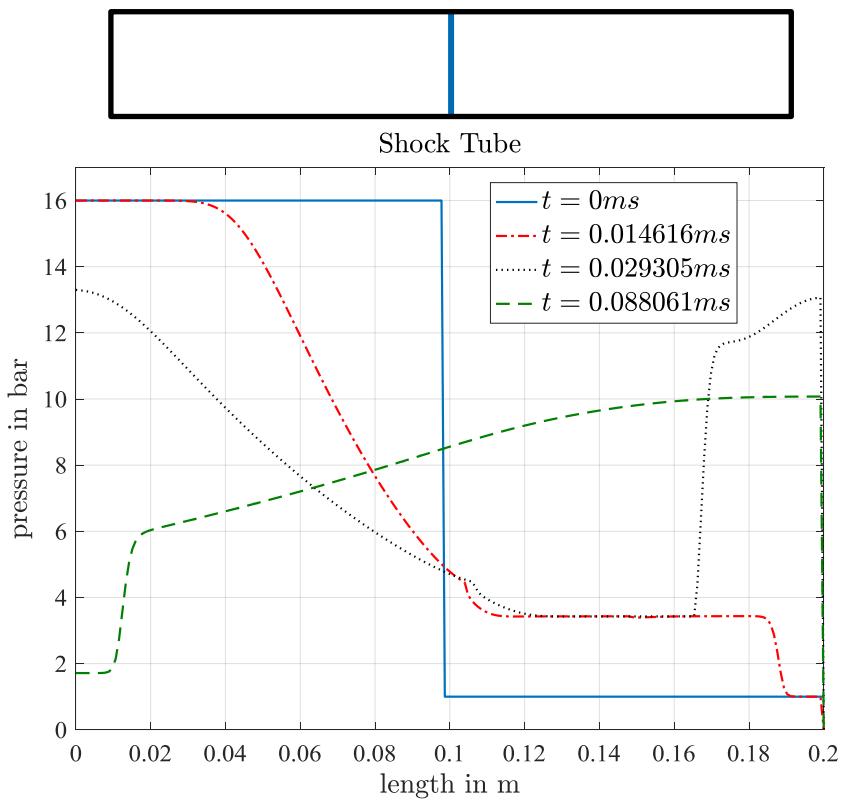


- Correct resonance pattern
- Lower magnitude

Shock Tube Theory

1D Pipe

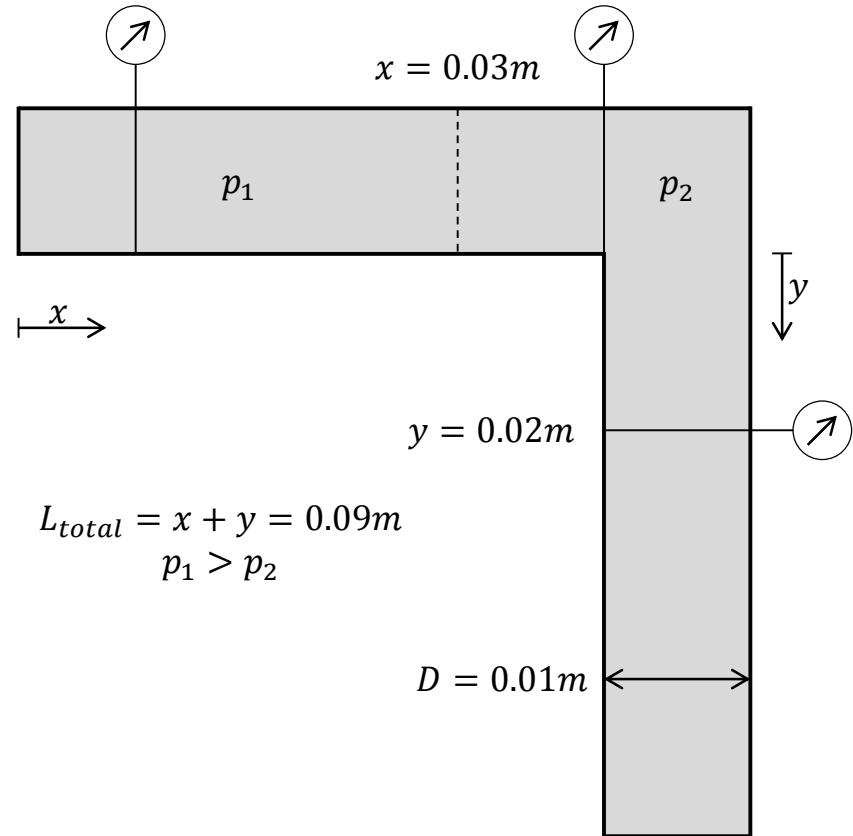
Sod's Shock Tube Test



2D Elbow

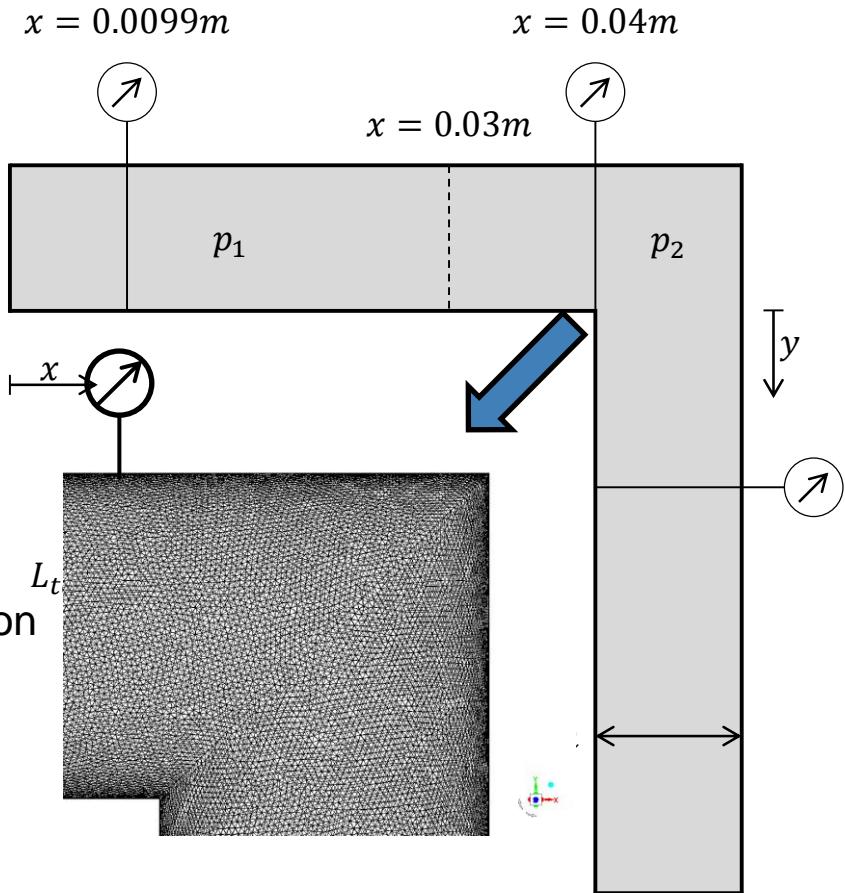
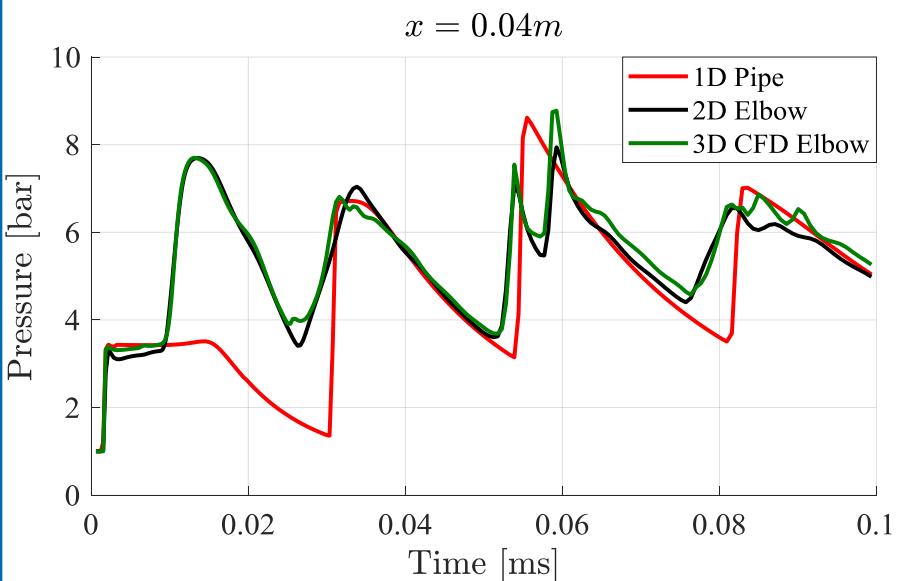
$$x = 0.0099m$$

$$x = 0.04m$$



Shock Tube Theory

Comparison to CFD



- Partial reflection corresponds to CFD solution
- Phase and amplitude accurate
- Deviation 2D elbow 7%
- Deviation 1D elbow above 20%

1 Finite Volume Fundamentals and Approach

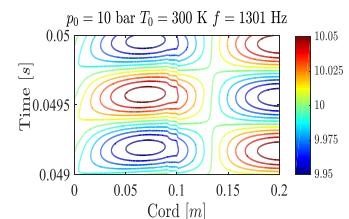
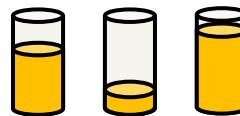
2 Analytical and Simulative Validation

3 Conclusion and Outlook

Conclusion and Outlook

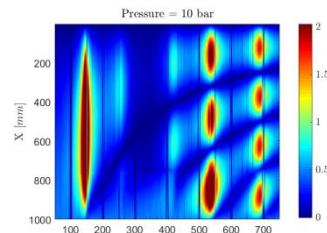
Conclusion

- Fundamentals of Finite Volume
- 2D/1D-Coupling
- Analytical and simulative validation



Outlook

- Acoustic reflection coefficient
- Curvature influence by source term modulation
- Test rig validation



Thank you for your attention!

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